Sheet 4

The line and bus data of power system shown in Fig (1) are given in per unit in Tables (1) and (2).

Find: The 4 Jacobians for NR method

Initial bus	Final bus	R	x_L	$\frac{y_C}{2}$
1	2	0.01008	0.0504	0.05125
1	3	0.00744	0.0372	0.03875
2	4	0.00744	0.0372	0.03875
3	4	0.01272	0.0636	0.06375

Table (1)

Bus No.	Bus voltage	P_G	Q_G	P_D	Q_D
1	$1\angle 0^0$	-	-	0.5	0.3099
2	$1\angle 0^0$	0	0	1.7	1.0535
3	$1\angle 0^0$	0	0	2.0	1.2394
4	$1.02\angle 0^0$	3.18	-	0.8	0.4958

Table (2)

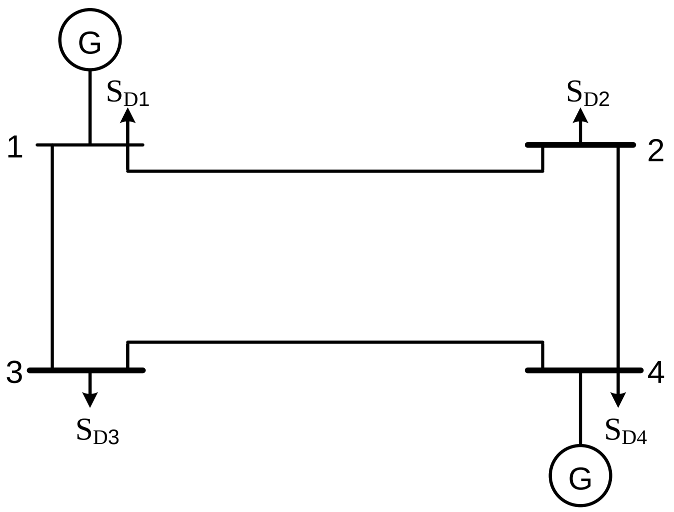

Figure (1)

ANS :

From data the bus types are taken as:

Bus No.	Bus type
1	Slack
2	Load
3	Load
4	Voltage control

$$Y_{bus} = \begin{bmatrix} 8.985 - j44.836 & -3.816 + j19.078 & -5.170 + j25.848 & 0 \\ -3.816 + j19.078 & 8.985 - j44.836 & 0 & -5.170 + j25.848 \\ -5.170 + j25.848 & 0 & 8.193 - j40.864 & -3.024 + j15.119 \\ 0 & -5.170 + j25.848 & -3.024 + j15.119 & 8.193 - j40.864 \end{bmatrix}$$

First Itteration before elimination

$$J_1^0 = \begin{pmatrix} 44.9260 & -19.0781 & -25.8478 & 0 \\ -19.0781 & 45.4429 & 0 & -26.3648 \\ -25.8478 & 0 & 41.2687 & -15.4209 \\ 0 & -26.3648 & -15.4209 & 41.7857 \end{pmatrix}$$

$$J_2^0 = \begin{pmatrix} 8.9852 & -3.8156 & -5.1696 & 0 \\ -3.8156 & 8.8818 & 0 & -5.1696 \\ -5.1696 & 0 & 8.1328 & -3.0237 \\ 0 & -5.2730 & -3.0842 & 8.5210 \end{pmatrix}$$

$$J_3^0 = \begin{pmatrix} -8.9852 & 3.8156 & 5.1696 & 0 \\ 3.8156 & -9.0886 & 0 & 5.2730 \\ 5.1696 & 0 & -8.2537 & 3.0842 \\ 0 & 5.2730 & 3.0842 & -8.3571 \end{pmatrix}$$

$$J_4^0 = \begin{pmatrix} 44.7460 & -19.0781 & -25.8478 & 0 \\ -19.0781 & 44.2290 & 0 & -26.3648 \\ -25.8478 & 0 & 40.4590 & -15.4209 \\ 0 & -25.8478 & -15.1185 & 42.3959 \end{pmatrix}$$

First Itteration after elimination

$$J_1^0 = \begin{pmatrix} 45.4429 & 0 & -26.3648 \\ 0 & 41.2687 & -15.4209 \\ -26.3648 & -15.4209 & 41.7857 \end{pmatrix} \qquad J_2^0 = \begin{pmatrix} 8.8818 & 0 \\ 0 & 8.1328 \\ -5.2730 & -3.0842 \end{pmatrix}$$

$$J_3^0 = \begin{pmatrix} -9.0886 & 0 & 5.2730 \\ 0 & -8.2537 & 3.0842 \end{pmatrix} \qquad J_4^0 = \begin{pmatrix} 44.2290 & 0 \\ 0 & 40.4590 \end{pmatrix}$$

The Jacobian Matrix J is:

$$J^0 = \begin{pmatrix} 45.4429 & 0 & -26.3648 & 8.8818 & 0 \\ 0 & 41.2687 & -15.4209 & 0 & 8.1328 \\ -26.3648 & -15.4209 & 41.7857 & -5.2730 & -3.0842 \\ -9.0886 & 0 & 5.2730 & 44.2290 & 0 \\ 0 & -8.2537 & 3.0842 & 0 & 40.4590 \end{pmatrix}$$

Second Itteration:

The Jacobian Matrix J is:

$$J^1 = \begin{pmatrix} 44.3749 & 0 & -25.6778 & 7.1397 & 0 \\ 0 & 39.7018 & -14.7736 & 0 & 5.9619 \\ -26.1256 & -15.1217 & 41.2473 & -4.1296 & -2.1828 \\ -10.3562 & 0 & 6.2998 & 43.0530 & 0 \\ 0 & -9.6597 & 3.8597 & 0 & 38.4642 \end{pmatrix}$$

Hints: *Jacobian matrix* at iteration k.

$$J_1^k = \left.\frac{\partial P_i}{\partial \delta_i}\right|_{\delta_i^k, V_i^k} \qquad J_2^k = \left.\frac{\partial P_i}{\partial V_i}\right|_{\delta_i^k, V_i^k} \qquad J_3^k = \left.\frac{\partial Q_i}{\partial \delta_i}\right|_{\delta_i^k, V_i^k} \qquad J_4^k = \left.\frac{\partial Q_i}{\partial V_i}\right|_{\delta_i^k, V_i^k}$$

The elements of Jacobian J_1:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i), \quad i = 1,2,3,\ldots\ldots,n$$

$$\frac{\partial P_i}{\partial \delta_j} = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i), \; i = 1,2,3,\ldots.,n, \; j \neq i$$

The elements of Jacobian J_2:

$$\frac{\partial P_i}{\partial V_i} = 2V_i Y_{ii} \cos\theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i), \quad i = 1,2,3,\ldots\ldots,n$$

$$\frac{\partial P_i}{\partial V_j} = V_i Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i), \; i = 1,2,3,\ldots.,n, \; j \neq i$$

The elements of Jacobian J_3:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i), \quad i = 1,2,3,\ldots\ldots,n$$

$$\frac{\partial Q_i}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i), \; i = 1,2,3,\ldots.,n, \; j \neq i$$

The elements of Jacobian J_4:

$$\frac{\partial Q_i}{\partial V_i} = -2V_i Y_{ii} \sin\theta_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n} V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i), \quad i = 1,2,3,\ldots\ldots,n$$

$$\frac{\partial Q_i}{\partial V_j} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i), \; i = 1,2,3,\ldots.,n, \; j \neq i$$